H.L.L. Busard: Jordanus de Nemore, *De elementis arithmetice artis. A medieval treatise on number theory*. Part I: *Text and paraphrase*. Part II: *Conspectus siglorum and critical apparatus*. Stuttgart (Franz Steiner Verlag) 1991. 372+188 pp. Kart. DM 138. ISBN 3-515-05214-3. (Boëthius Bd. XXII, 1-2).

Since the mid-nineteenth century, Jordanus de Nemore has been recognized as one of the two most original mathematicians of the Latin thirteenth century. There has thus also been recurrent interest in his extensive *Elements of Arithmetic*, together with the *De ratione ponderis* (if this is indeed his) the only one of his works which was printed in the Renaissance (by Lefèvre d'Étaples, Paris 1496 and 1514). No modern edition, however, has been available, and Lefèvre d'Étaples' presentation of the published work as "Jordanus' Arithmetical Elements, with Lefèvre d'Étaples demonstrations" has given rise to considerable doubts concerning the character of the original work—doubts not settled by the scant reports of the content of manuscripts to be found in the literature.

Alongside of his impressive effort to make the twelfth-century Latin versions of the Euclidean *Elements* available (five versions to date, one together with Menso Folkerts), Hubert L. L. Busard has now offered a critical edition of these thirteenth-century arithmetical *Elements* to the students of Medieval mathematics. Needless to say to anybody who has had the pleasure to use his earlier critical editions, the two volumes now at our disposal are of high quality.

Except in cases where the other manuscripts correct obvious slips of the copyist's pen, Busard's text follows the words as well as the diagrams of the Paris manuscript BN lat. 16644—not only the earliest and the "best" according to internal criteria but also prepared directly for Richard de Fournival, who appears [reviewer's comment] to have been in contact either with Jordanus himself or with direct disciples of his. Except for orthographic variations and substitutions of *ergo* for *igitur* and vice versa, the variant readings of 19 other manuscripts are listed in the critical apparatus (the added propositions and alternative proofs found in and characterizing the secondary manuscript families II and III, however, are given in an appendix to the text edition). Of the five additional manuscripts which are known, one is heavily damaged by water and the others contain only fragments.

No translation *stricto sensu* nor any detailed mathematical commentary are given, but a paraphrase in English and letter symbolism (largely Jordanus' own) will be a valid substitute for both to readers possessing elementary amiliarity with Medieval Latin.

Furthermore, an initial 24 pages' "summary" (much more indeed) give an overview of the character and detailed content of each of the ten books; being followed by an eight-page long tabulation of possible sources and users of the single propositions, this summary together with the careful text edition and the paraphrase represents a pair of broad shoulders on which any further research on Jordanus' *Arithmetic* may stand

firmly.

The editor himself takes deliberate care to draw only such conclusions which follow directly from the text, and even these are set forth *sotto voce;* instead of stating bluntly that colleagues (living as well as deceased) are wrong, Busard prefers to conclude repeatedly that he "does not understand" a point of view which he has just disproved effectively.

One noteworthy observation is that Jordanus' work follows the style of the Adelard II *Elements.* This holds both for the style of the proofs (often only a sketched proof idea) and in making proofs precede enunciations (the partial deviation from this order in certain manuscripts is clearly secondary; that it is, none the less, not observed in the edition may be for the good reason that the wording of many proofs—clearly, e.g., II.ix—presupposes the exact formulation of the enunciation).

As pointed out by Busard, the proofs given in the *editio princeps* are quite different— longer, more detailed, at times somewhat pedantic implementations [reviewer's opinion] of the original proof idea. They are obviously of Lefèvre d'Étaples' own making, which clarifies the meaning of his presentation as quoted above. Fortunately for those of us who believed in Curtze's statement that they were not, Lefèvre d'Étaples uses Jordanus letter-formalism precisely in the way it was introduced, only more thoroughly, re-presenting numbers—and, e.g. in V.viii, even ratios—by a single letter but using no hint of algebraic formalism. Only Lefèvre's pervasive use of numerical illustrations and the style of certain diagrams misrepresent the original decisively.

Also important is the observation made in passing (p. 19) that "book V looks like an algorism of ratios" though differing from Oresme's *Algorismus proportionum* in its exclusive treatment of ratios between numbers (thus rational ratios). The scope of Jordanus' algorism is illustrated by propositions V.xi and V.xiv which show, respectively, that even though no number can be divided into extreme and mean ratio, numbers can be found which approach this division to any degree required; and that any ratio, even though it may not allow exact bisection, can be composed from two ratios whose difference is less than any given ratio ("composition" and "difference" to be understood in the likeness of the "addition" of and difference between musical intervals).

Jordanus' text as we now possess it, together with Busard's listing of parallels to the single propositions, should definitively eliminate the notion that it is nothing but a continuation of the Nicomachean-Boethian tradition (cf. p. 23). The large number of propositions for which no precursors are known also demonstrates that more than a mere synthesis of the Euclidean and the Boethian tradition (providing the latter with proofs) is involved: Basing himself on the concepts, propositions and techniques of these two traditions and reordering the material, Jordanus creates a sequence of new, partially connected theoretical structures calling for their own extensions of the inherited material—as exemplified by the "algorism of ratios" of book V.

One final observation struck the reviewer during the reading of Jordanus' text: Its rhetorical profusion, rather unusual in mathematical texts from Jordanus' as well as other epochs. Instead of closing his proofs by a stereotype phrase in the likeness

of QED (e.g., "et hoc voluimus demonstrare", recurring in Gerard of Bruxelles' *Liber de motu*), Jordanus uses an dazzling array of variations, never repeating himself from one proof to the next. This rhetorical inventiveness reminds of Jordanus' introduction to his short set of algorism treatises, and suggests that these and the *Arithmetic* might be chronologically close (and probably early in Jordanus' career). The *Liber philotegni* conserves only traces of the rhetorical variation and might thus be late; both the *Elementa super demonstrationem ponderum* and the *Liber de ponderibus* remind of the *Arithmetic* in this respect. The short treatise *De proportionibus* ascribed on one occasion to Jordanus is fully stereotypized and thus—so it looks—after all falsely ascribed.

Further elaboration of these ideas would require thorough internal and comparative text analysis, and they may eventually prove unfounded or undecidable. Whatever the outcome of such scrutiny, however, the very possibility to formulate the hypotheses highlights the merit of the present edition, as of any good edition of an important but badly known work: not only informing us about the thought of the past and killing off prejudice and mistake, it also allows us to start afresh on new and unexpected exploration. For all of which the editor deserves our gratitude.

<div align="right">Jens Høyrup</div>